

## Lesson 6

$Y=mx+b$ : This is the equation of a line.

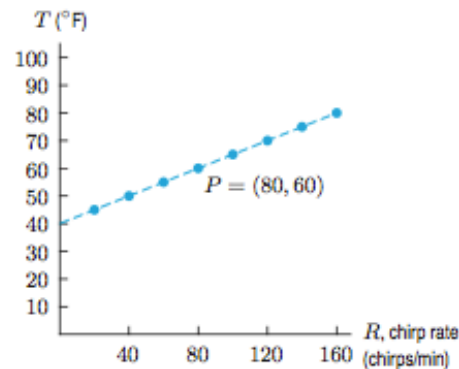
$M$ = slope

$B$ = y-intercept

Express relationship between two quantities

**Table 1.1** Chirp rate and temperature

| $R$ , chirp rate<br>(chirps/minute) | $T$ , predicted<br>temperature ( $^{\circ}\text{F}$ ) |
|-------------------------------------|---|
| 20                                  | 45  |
| 40                                  | 50  |
| 60                                  | 55  |
| 80                                  | 60  |
| 100                                 | 65  |
| 120                                 | 70  |
| 140                                 | 75  |
| 160                                 | 80  |



**Figure 1.1:** Chirp rate and temperature

- **Formula:** A formula is an equation giving  $T$  in terms of  $R$ . Dividing the chirp rate by four and adding forty gives the estimated temperature, so:

$$\underbrace{\text{Estimated temperature (in } ^{\circ}\text{F})}_T = \frac{1}{4} \cdot \underbrace{\text{Chirp rate (in chirps/min)}}_R + 40.$$

Rewriting this using the variables  $T$  and  $R$  gives the formula:

$$T = \frac{1}{4}R + 40.$$

Let's check the formula. Substituting  $R = 80$ , we have

$$T = \frac{1}{4} \cdot 80 + 40 = 60$$

which agrees with point  $P = (80, 60)$  in Figure 1.1. The formula  $T = \frac{1}{4}R + 40$  also tells us that if  $R = 0$ , then  $T = 40$ . Thus, the dashed line in Figure 1.1 crosses (or intersects) the  $T$ -axis at  $T = 40$ ; we say the  $T$ -intercept is 40.

When we use a function to describe an actual situation, the function is referred to as a **mathematical model**. The formula  $T = \frac{1}{4}R + 40$  is a mathematical model of the relationship between the temperature and the cricket's chirp rate. Such models can be powerful tools for understanding phenomena and making predictions. For example, this model predicts that when the chirp rate is 80 chirps per minute, the temperature is  $60^{\circ}\text{F}$ . In addition, since  $T = 40$  when  $R = 0$ , the model predicts that the chirp rate is 0 at  $40^{\circ}\text{F}$ . Whether the model's predictions are accurate for chirp rates down to 0 and temperatures as low as  $40^{\circ}\text{F}$  is a question that mathematics alone cannot answer; an understanding of the biology of crickets is needed. However, we can safely say that the model does not apply for temperatures below  $40^{\circ}\text{F}$ , because the chirp rate would then be negative. For the range of chirp rates and temperatures in Table 1.1, the model is remarkably accurate.

In everyday language, saying that  $T$  is a function of  $R$  suggests that making the cricket chirp faster would somehow make the temperature change. Clearly, the cricket's chirping does not cause the temperature to be what it is. In mathematics, saying that the temperature "depends" on the chirp rate means only that knowing the chirp rate is sufficient to tell us the temperature.

**Example:** Solve for x in the following equation

$$2x - 4 = 10$$

Add 4 to both sides of the equation:  $2x = 14$

Divide both sides by 2:  $x = 7$

**The answer is  $x = 7$ .**

Check the solution by substituting 7 in the original equation for x. If the left side of the equation equals the right side of the equation after the substitution, you have found the correct answer.

$$2(7) - 4 = 14 - 4 = 10.$$

**Example:** Solve for x in the following equation

$$5x - 6 = 3x - 8$$

Subtract 3x from both sides of the equation:  $2x - 6 = -8$

Add 6 to both sides of the equation:  $2x = -2$

Divide both sides by 2:  $x = -1$

**The answer is  $x = -1$**